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# Usage of Eulerian and Hamiltonian Graph in Pandemic Situation

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## Abstract

*The existence of Euler and Hamiltonian graph make it easier to solve real life problem. During the time of pandemic "Covid -19" it is very essential for each one of us to be vaccinated. Vaccination is done in the hospitals by using Eulerian and Hamiltonian graph not only prevent people from infecting but also increase the speed of vaccination. In this paper, authors have discussed the use of graph theory to effectively handle the covid – 19 pandemic situations in the allocation of patients to appropriate hospitals. The work done in this paper, may be utilized for the hospital – patient management as well as for the study by upcoming researchers to utilize the graph theory in any complicated situations.*

## Keywords

*Graph, Eulerian, Hamiltonian, path, circuit, pandemic, hospital.*

## 1. Introduction

In Mathematics, Graphs play a very important role. Graph is the representation of problems or application in graphical format. With the help of graphs, one can understand and solve the real-life problems. Graphs not only help in understanding, but it also makes our study easier. Graphs consist of edges and vertices. We generally represent graph as an ordered pair of vertex and edge i. e. G (V, E).

Vertices are those point which connect with each other through line called edge. It is represented by V. The vertex in graphs also known as points or nodes of a graph. Edges are the connecting lines which join 2 vertices together. It is represented by E. Vertex are said to be adjacent if they have common edge between them.

There are different types of graphs in graph theory-

- Finite graph: if a graph has finite number of vertices and edges then the graph is said to be finite.
- Trivial graph: if a graph has only one vertex and no edges then it is said to be trivial graph.
- Subgraphs: if vertices and edges of H are contained in the vertices and edges of graph G.

- Complete graph: A graph is said to be complete if every vertex is connected to every other vertex.
- Regular graph: a graph is said to be regular if every vertex has the same degree.
- Bipartite graph: if its vertices can be divided into 2 subsets p and q such that each edge of graph G connects a vertex of P to a vertex of Q.
- Isomorphic graph: if there exists a one to one correspondence such that {u, v} is an edge of graph if and only if a {f(u) , f(v)} is an edges in g*.
- Homomorphic graph: if we can obtain another graph by adding the vertices to the given graph.
- Eulerian graph: A connected graph G is said to be Euler graph, if it follows Eulerian circuit. In other words, if there exist a closed trail which includes every edge of the graph G.
- Hamiltonian graph: A graph is said to be Hamiltonian if it follows Hamiltonian circuit.

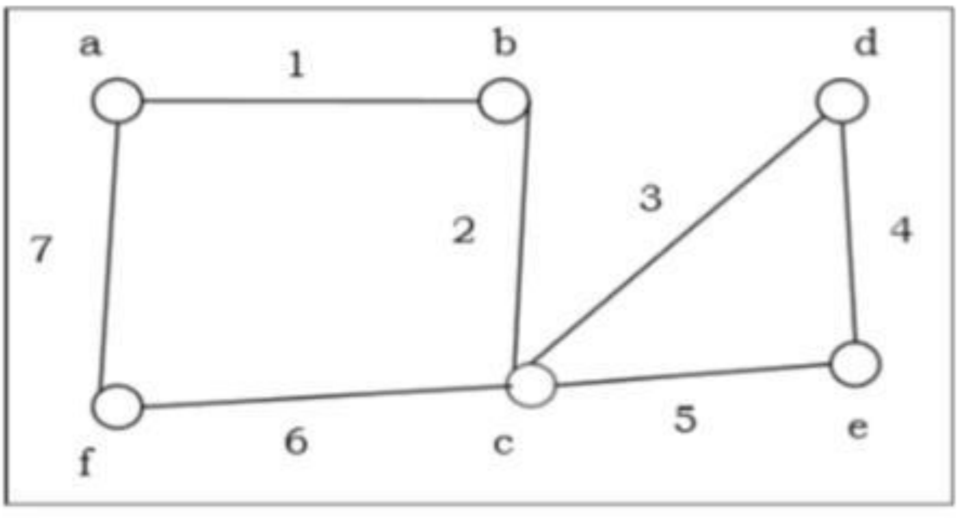


Figure 1. Eulerian Graph, Eulerian Path, Eulerian Circuit

A connected graph G which contains a Euler cycle is called Euler graph. A graph is said to be connected when there is a path between every pair of vertices. A closed walk which visits every edge of the graph exactly once is called Eulerian circuit or Eulerian cycle. A path that uses every edge of a graph exactly once is called Eulerian path. A Euler path starts and ends at different vertices. An open walk is that which visit every edge of the graph exactly once. A graph which contains an open euler walk is called semi eulerian graph.

A graph is said to be Euler if it follows two conditions:

- If it has same start and ends point.
- If its path does not repeat any edge i.e. if it visit every edge just once.
- The vertex has restriction, it may or may not repeat.

Direct method for Euler graph: If a graph is Euler, then every vertex in a graph has even degree. If a graph is semi-Euler graph, then, it has all vertex of even degree expect 2 vertices.

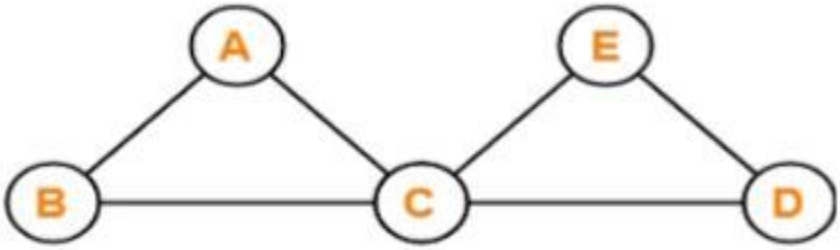


Figure 2. Euler Graph

In the given figure, the path is connected. If we considered B as the start vertex and start covering the rest of the vertex like {B---A---C---E---D---C---B} and the graph cover the following edges like { BA---AC---CE---ED---DC---CB}. The start and the end point remain same i.e. B in the given figure. Every edge of the given graph covers exactly once. Hence given graph is Euler graph. By direct method deg(A)=2, deg(B)=2, deg(C)=2, deg(D)=2, deg(E)=2. Since every vertex has even degree, so it is Euler graph.

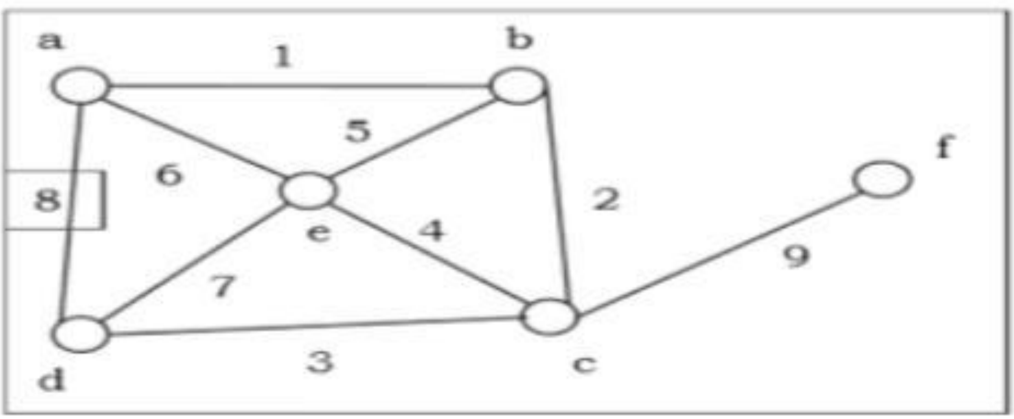


Figure 3. Hamiltonian graphs, Hamiltonian path and circuit

A connected graph which contains a Hamiltonian circuit/cycle is called Hamiltonian graph. If in a connected graph, there exists a path that contains all the vertices of the graph is called Hamiltonian path. A Hamiltonian path which starts and ends at the same vertex is called a Hamiltonian circuit [1]. For a graph to be Hamiltonian it should follow 2 conditions:

- The start and the end vertex point should be same.
- Every vertex should visit exactly once.
- There is no restriction on edges i.e. it may / may not repeat.

## Direct method for Hamiltonian graph

A complete graph which has more than 2 vertices is always a Hamiltonian graph. A cycle graph is always a Hamiltonian graph. Other graph can also be Hamiltonian graph for that we have to show the conditions mentioned above.

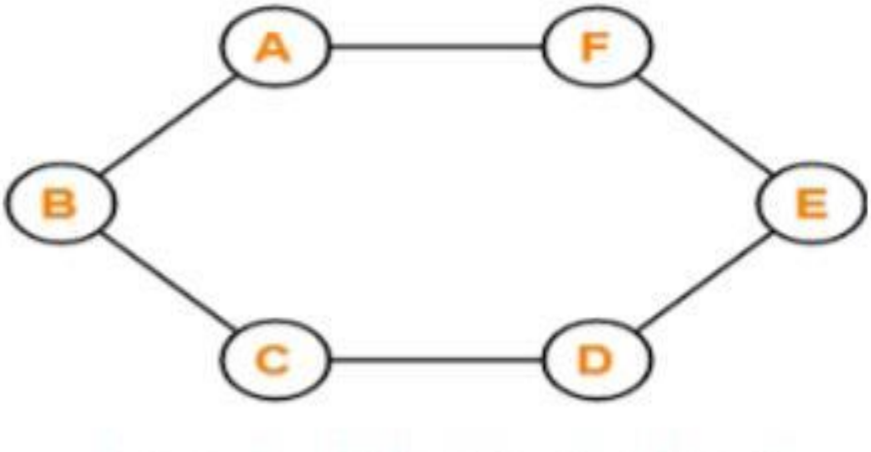


Figure 4. Example of Hamiltonian graph

In the given figure, the path is connected. If we considered B as the start vertex and start covering the rest of the vertex like {B---A---F---E---D---C---B} and the graph cover the following edges like {BA---AF---FE---ED---DC---CB}. The start and the end point remain same i.e. B in the given figure. Every vertex of the given graph covers exactly once [2]. Hence, the given graph is Hamiltonian graph. By Direct method, A given graph is a cycle and we know a cycle graph is always Hamiltonian. Hence, the given graph is Hamiltonian. Condition for a graph to be Eulerian and Hamiltonian:

- Starting and ending vertex remain same.
- Each edge of the graph should use exactly once.
- Each vertex of the graph should use exactly once.

## 2. Application of Eulerian and Hamiltonian graph in pandemic situation

### 2.1. Application of Euler and Hamiltonian graph in vaccination

Vaccination is a simple, easy, and effective way of protecting people from harmful diseases before it come into contact with us. Recently, the vaccination against covid-19 is in news [3]. One question arises in my mind – is it possible to apply graph theory in it? The answer I got after my research work is yes, we can. Then again question arises- How? The answer I got is by using Eulerian and Hamiltonian graph.

Vaccination is done in the hospitals and in covid-19 vaccination two doses is given to a single person i.e. first dose and second dose after 84 days. If we apply the divided the total hospitals of a particular city into 4 parts and apply EULERIAN and HAMILTONIAN graph in each of it, then we can easily increase the speed of vaccination. Suppose we have total "W" hospitals

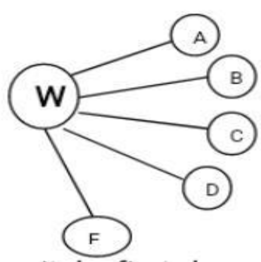


Figure 5. Categorization of the hospitals w.r.to age limit

in particular city, as shown in the figure 5. Then we divide those W hospitals in 4 parts (on the basis that the route/path of the hospitals are different)- ‘A’,’B’,’C’,’D’,’E’ [4].

In ‘A’ Type hospitals (figure 6), first dose of vaccine will be provided to 45+ candidates. The candidates are requested to follow Eulerian and Hamiltonian path in order to receive first dose. As shown in the figure.

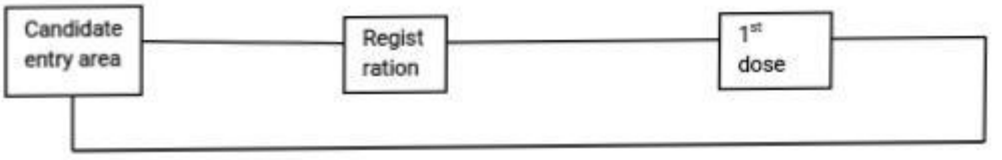


Figure 6. Vaccination process for Type A hospitals

As we able to see that every point or vertex is covered exactly once and every edge or path from one point to another is also covered exactly once hence the path followed is Eulerian and Hamiltonian path and thus the graph formed is by applying the Eulerian and Hamiltonian graph [5]. Similarly, in ‘B’ type hospitals (figure 7), first dose of vaccine will be provided to 18+ candidates. The candidates are requested to follow Eulerian and Hamiltonian path in order to receive first dose.

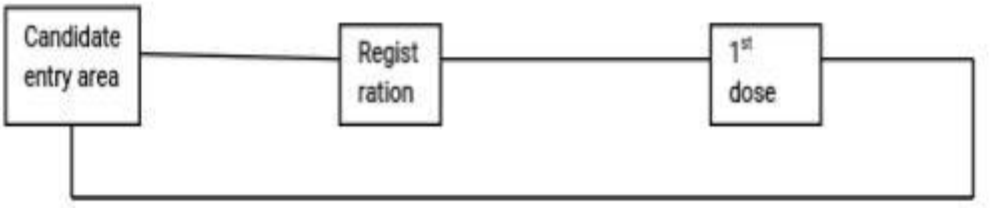


Figure 7. Vaccination process for Type B hospitals

Every point or vertex is used exactly once, and every edge or path is also used once hence Eulerian and Hamiltonian path is followed by every candidate and thus the graph so formed is Eulerian and Hamiltonian graph [6]. In ‘C’ type hospitals, second dose of vaccine will be provided and the candidate who visit option 1st type or option 2nd type hospitals, will visit here after 84 days of their first dose. The candidates here also follow Eulerian and Hamiltonian path in order to receive the second dose (figure 8). Thus, the graph so formed is Eulerian and Hamiltonian graph [7].

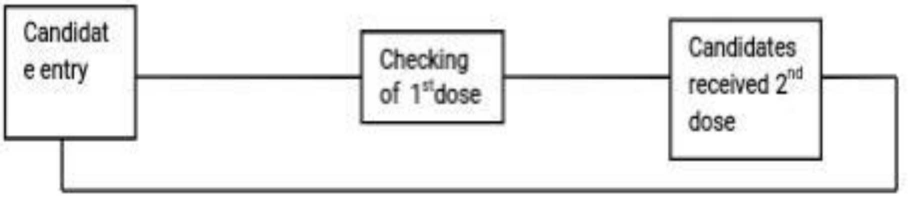


Figure 8. Vaccination process for Type C hospitals

In ‘D’ type hospitals, covid-19 +ive patients are treated. The candidate visit to this hospital may or may not visit to ‘A’ or ‘B’ type hospital. In ‘E’ type hospitals, covid-19 test should be done. The candidate whose report will be positive should refer to ‘D’ type hospital and also should advise to disconnect himself from rest of the paths. Application of Euler and Hamiltonian graph in preventing infection from spreading

In above text, we have divided the total 'W' hospitals into 4 types i.e. we stop visiting of random candidate into any hospitals for vaccination of 1st &2nd dose, for covid-19 treatment and for covid-19 test. By disconnecting the path (and providing Eulerian and Hamiltonian path for specific need) of candidate for vaccination from the candidate for covid-19 treatment not only help people for easily vaccinated but also prevent people from coming in contact with infected person.
By applying the Eulerian and Hamiltonian path in each type hospitals, also secure/ prevent people from coming in contact with each other [8].

## Condition before using the Eulerian and Hamiltonian path

Suppose the person "N" feel ill and come for covid-19 test, but due to no specific path, he went in the vaccination area. There the person "N" met another person to ask for the covid-19 test area. When he was tested positive. All persons who came in his contact also tested positive and this process remain continue, as depicted in figure 9.

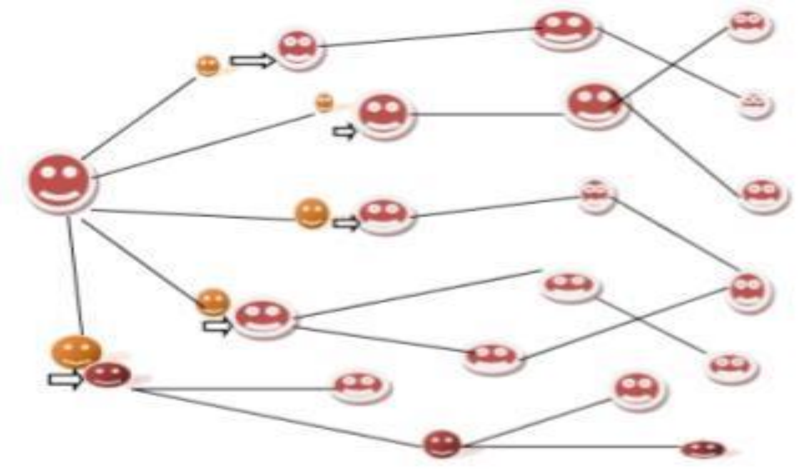

Figure 9. Spreading of infection

The above diagram shows how single infected person infect others, as depicted in figure 9. This spreading of infection can be prevented if we provide the separate place for covid-19 test. The best way to stop spreading of covid-19 in hospitals, vaccination center and tested area is to disconnect their path from each other. This can be achieved by simplifying disconnecting some paths and by follow the Eulerian and Hamiltonian path in each hospital [9].

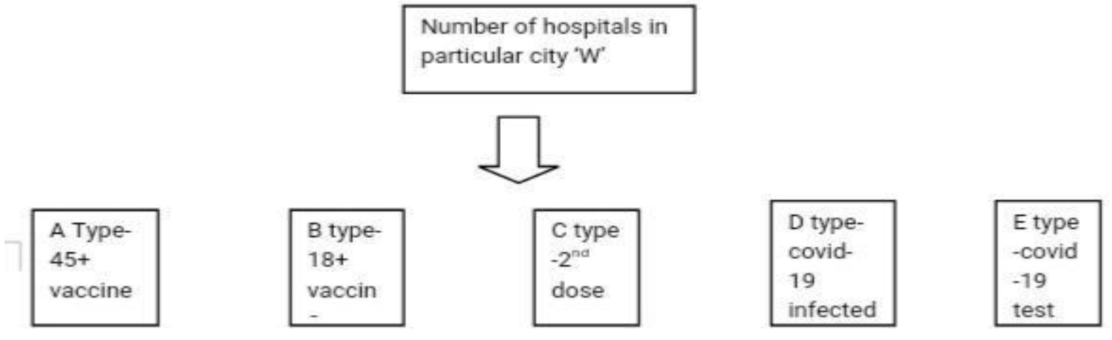


Figure 10. Simplification of disconnecting paths to follow the Eulerian and Hamiltonian path in each hospital

### Condition after using the Eulerian and Hamiltonian path

Suppose the person "N" feel ill and come for covid-19 test, as it is specified that testing for covid-19 is in hospital of type "D". The person "N" will visit there and no one else get infected from him in case he will tested positive. By disconnecting five paths from each other, we can easily safe millions of live. In other words, we can say by dividing the single root vertex into 5 other vertex and by dividing single edge into five different edges, we can prevent infection from spreading [10]. Each type of hospital follows the Eulerian and Hamiltonian path i.e. no person is allowed to repeat same room again and to skip any room or process (no vertex repeat itself and every vertex visit once, every edge is in path exactly once). Now, to understand Eulerian and Hamiltonian path in each type of hospitals.

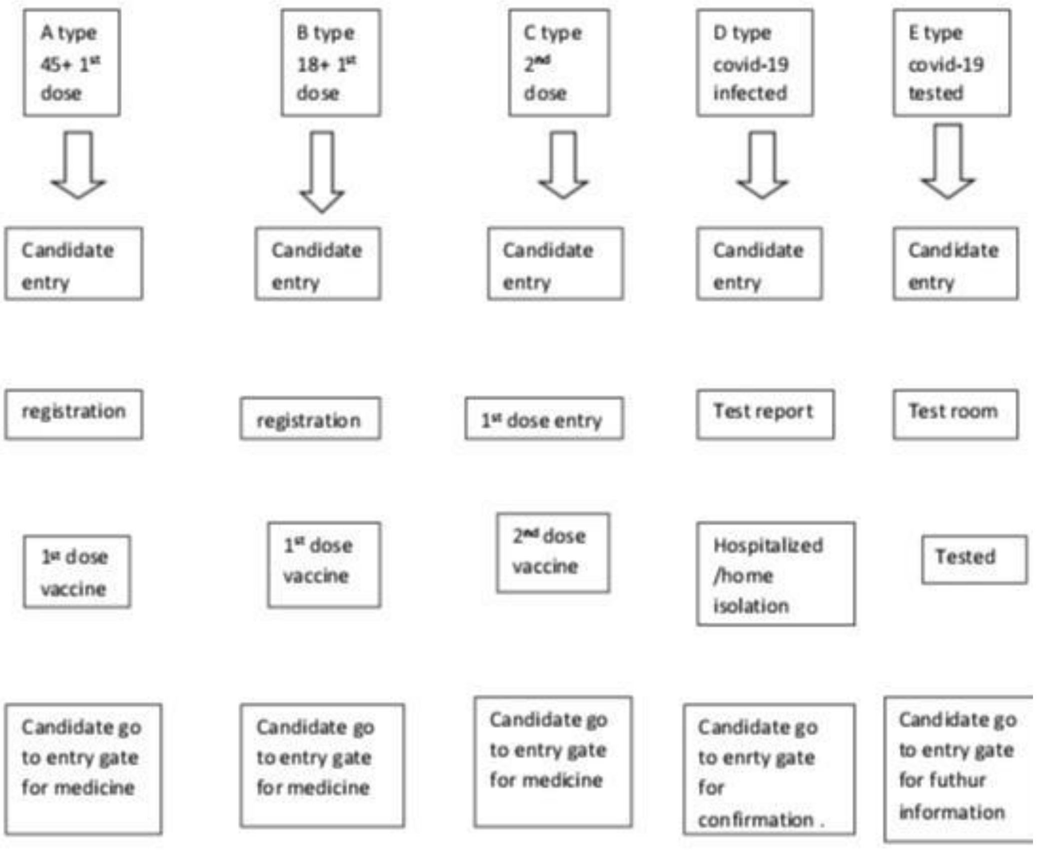


Figure 11. Preventing infection to spread using Eulerian and Hamiltonian graph

Clearly from the above graph we can understand that how with the help of Eulerian and Hamiltonian graph, we can prevent infection from spreading.

## 3. Conclusion

Graph is the basic of graph theory. It helps to represent a real-life problem in graphical format. In other words, graph is a set of points or nodes or vertices that are joined by edges such that each edge ends and starts at one of the vertices. The main aim of this research paper is to explain Euler and Hamiltonian graph and their application. We discussed the application of Euler and Hamiltonian theory to solve- Vaccination problem of 1st dose above 45 age, vaccination problem of 1st dose above 18 age, vaccination problem of 2nd dose, treatment of covid-19 infected person problem, test of covid-19 problem and prevent people to get infected from another infected person. The existence of Euler and Hamiltonian graph make it easier to solve real life problem. The Euler and Hamiltonian path allow the candidate to visit exactly once at any point, that avoids more interaction of people with one another.

## References

[1]. V. Sutaria, “A Hamiltonian and Eulerian cycles”, International Journal of Mathematics Trend and Technology, vol. 3, iss. 5, pp. 208-212, 2016.

[2]. A. Kumar and SHUATS, “A study on Euler Graph and it’s applications,” Int. j. math. trends technol., vol. 43, no. 1, pp. 9–15, 2017.

[3]. H. Ahmed, “Graph routing problem using Euler’s theorem and its applications,” Engineering Mathematics, vol. 3, no. 1, p. 1, 2019.

[4]. H. Fleischner, “(Some of) the many uses of Eulerian graphs in graph theory (plus some applications),” Discrete Math., vol. 230, no. 1–3, pp. 23–43, 2001.

[5]. M. S. Rahman and M. Kaykobad, “On Hamiltonian cycles and Hamiltonian paths,” Inf. Process. Lett., vol. 94, no. 1, pp. 37–41, 2005.

[6]. F. Keshavarz-Kohjerdi and A. Bagheri, “Hamiltonian paths in some classes of grid graphs,” J. Appl. Math., vol. 2012, pp. 1–17, 2012.

[7]. Ł. Waligóra, “Application of Hamilton’s graph theory in new technologies,” World Scientific News, vol. 89, pp. 65–76, 2017.

[8]. S. A. L. T. Cherin Monish Femila, “Hamiltonian cycle and Hamiltonian path decomposition of fan graphs,” International Journal of Scientific Research in Mathematical and Statistical Sciences, vol. 6, no. 2, pp. 207–211.

[9]. W. Alhalabi, O. Kitanneh, A. Alharbi, Z. Balfakih, and A. Sarirete, “Efficient solution for finding Hamilton cycles in undirected graphs,” Springerplus, vol. 5, no. 1, p. 1192, 2016.

[10]. K. ChandraBora and B. Kalita, “Particular Type of Hamiltonian Graphs and their Properties,” Int. J. Comput. Appl., vol. 96, no. 3, pp. 31–36, 2014.