Journal of Applied Science and Education, 2021,
Vol. 01, Iss. 01, S. No. 005, pp. 1-4

Derivatives: A Comprehensive Study of Rate of Change

Ambrish Kumar Pandey[1], Samkach Singh[2]

[1,2]Department of Mathematics, Amity School of Applied Science, Amity University, Lucknow
[1]johan.pandey@gail.com

How to cite this paper: A. K. Pandey and S. Singh, "Derivatives: A Comprehensive Study of Rate of Change," *Journal of Applied Science and Education (JASE),* 1(1), 005, pp 1-4, 2021.

http://doi.org/10.54060/JASE/001.01.005

Received: 02/10/2021
Accepted: 03/11/2021
Published: 11/11/2021

Abstract

In mathematics, derivatives are the rate of change of a function with respect to a variable, and they are necessary for answering complex mathematical problems and differential equations. In this paper, a detailed study on the use and concept of derivative that how it comes into existence, how it can be used to calculate the differential coefficients of a function at a particular point in an effective manner, and what are the applications of the rate of change of functions in mathematics as well as in real-life situations is presented. This study will help to understand the in-depth concept of calculus to the new researchers and students of mathematics.

Keywords

Solving Computational Problems, Differential equation Component, Derivative functions.

1. Introduction

A rate of modification of a function, like y = f(x), is determined by measuring the amount of modification in y in relation to a modification in x. It is commonly nicknamed "rise over run"---the amount of vertical change ("rise") divided by the amount of horizontal change ("run"). This gives the slope of line drawn between the initial and final points (x1, y1) and (x2, y2). If the function is not linear, however, the computed rate of change is only an approximation [1]. Since the curve could have significant changes of shape (and therefore slope) between initial and the final points, a simple slope calculation is likely to miss the important information about curve [1].

To improve our knowledge of the behavior of the curve, we differentiate the function---we take the derivative. This involves finding the slope over a small region of the curve---so small as to be infinitesimal [2]. The effect is to find the slope at a single point, instead of between the usual two points.

The formal way to do this is with the "difference quotient":

$$f'(x) = f(x + h) - f(x) / h.$$

Here, "h" is a slight modification in the x-value, and f(x + h) is the corresponding change in the y-value. The true value of the derivative is found by decreasing h to zero, thereby reducing to zero the error in the approximation. The terms in the fraction are algebraically manipulated so as to be able to cancel a factor of h in both numerator and denominator. The remaining function is f(x), [3] which is the derivative function. The slope of the line that is tangent to the curve at the point is its value at x. This slope represents the current fluctuation of the primary function [4].

2. Application of Derivatives in Mathematics

Different applications of derivatives may be found not only in mathematics and real life, but also in other domains such as design, science, manufacture, physics, and so on [4].
In mathematics, derivatives are useful for a variety of things, including:

- Rate of modification of a Amount
- Decreasing and Increasing Functions
- Normal and Tangent to a Curve
- Maximum and Minimum Values
- Newton's Method
- Straight Estimates

The derivative is the pace at which one measure alterations in relation to another. The speed of change of a function is described as dy / dx = f(x) = y' in terms of functions. Derivatives are a notion that has been employed on both a local and huge scale. The notion of derivatives is applied in a variety of ways, including temperature change, rate of growth of size and shapes of an item relaying on circumstances, and so on.

3. Rate of Change of a Quality

This is the most common and significant use of derivatives. For example, we may use the derivatives form dy/dx to examine the speed of adjustment of the capacity of a cube about its reducing sides. Where dy is the rate of change of the cube's volume, and dx is the rate of growth of the cube's edges [5].

4. Increasing and Decreasing Function

Researchers utilise derivatives to determine whether a given function is rising, decreasing, or constant, for example, on a graph. If f is a continuous function in [p, q] and a differentiable function in the open interval (p, q),

- f is increasing at [p, q] if $f'(x) > 0$ for each $x \in (p, q)$
- f is decreasing at [p, q] if $f'(x) < 0$ for each $x \in (p, q)$
- f is constant function in [p, q], if $f'(x)=0$ for each $x \in (p, q)$

5. Tangent and Normal to a Curve

Standard is the orthogonal to that tangent, where tangent is the line that touches the curve at a point but does not cross it.

Allow the tangent to intersect the curve at P (x1, y1)

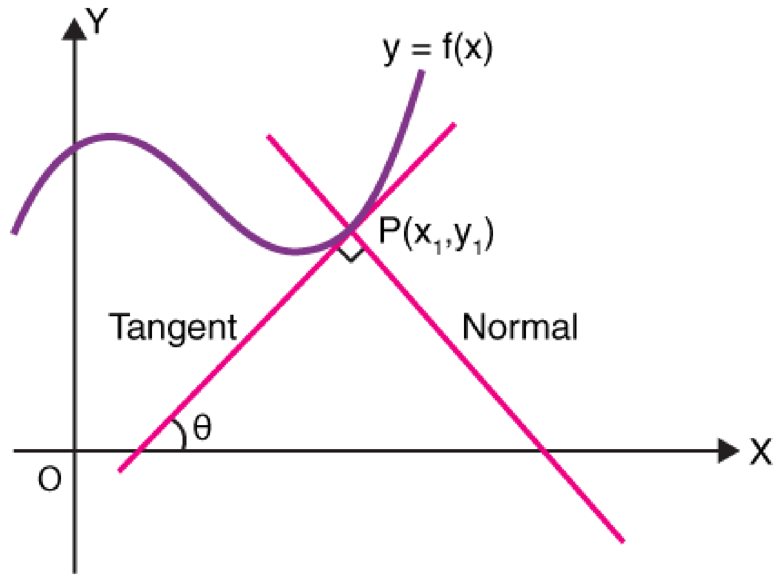

Fig 1. Tangent meet the Curve

Now write the straight-line formula that passes through a point with gradient m as:

$$y - y1 = m(x - x1)$$

At the point P(x1, y1), the gradient of the tangent to the curve y = f(x) is provided as in the preceding formula:

$$dy/dx \text{ at } P(x1, y1) = f'(x)$$

As a result, the tangent to the curve formula at P(x1, y1) may be expressed as

$$y - y1= f'(x1)\ (x - x1)$$

The normal to the curve formula is as follows:

$$y - y1= [-1/ f'(x1)]\ (x - x1)$$

Or

$$(y - y1)\ f'(x1) +(x-x1) = 0$$

6. Maxima and Minima

To calculate the highest and lowest point of the curve in a graph or to know it turning point, the derivative function is used [5].

- If f(x) ≤ f(a) for every x in the domain when x = a, so f(x) has an Absolute Maximum value, and the position a is the location where f(x) reaches its maximum significance.
- If for each x in some open period (p, q), f(x) ≤ f(a), then f(x) has a Comparative Highest benefit.
- When x=a and f(x) ≥f(a) for each and every x in the region, f(x) has an Extreme Minimal value, and position a is the location where f(x) has its minimal value.
- If f(x) ≥ f(a) for all x in any open range (p, q) where x = a, then f(x) has a Comparative Minimal value.

7. Application of Derivatives in real life

The principles for these applications will be better grasped by solving application of derivatives questions.

- Graphs are used to determine the profit and loss of a firm.
- Checking the temperature fluctuation.
- To calculate the speed or space travelled in kilometers per hour, miles per hour or other units.
- Several formulas in physics are derived using derivatives.
- In the field of seismic waves, it's common to look for the earthquake's magnitude scale.

8. Conclusion

The derivative is a formal way to discuss rate of change. Asking for the derivative of x with respect to y (dx/dy) is just a fancy way of asking how x changes when y changes. It is changing all the time. And if someone want to understand how the world is changing, the derivative is a necessary mathematical tool for doing that. This is one of the most common types of derivatives in applications of math: a derivative with respect to time. Say anybody's position at a particular moment is x. The derivative with respect to time asks how his position changes with respect to time. This is a fancy way of asking how fast one is going. Say one have an equation that tell him where he'll be at every moment in time. Take the derivative of that function, and one gets a new function that tells how fast one'll be going at each moment. Calculus is critically applicable in a variety of subjects: physics, economics, engineering, statistics, finance, some parts of biology, and so on.

References

[1]. J. Cadena, B. Travis, and S. Norman, "An evaluation of reform in the teaching of calculus," *Mathematics and Computer Education*, vol. 37, no. 2, pp. 210–220, 2003.

[2]. H. M. Nahikian, W. A. Granville, P. F. Smith, and W. R. Langley, "Elements of the differential and integral calculus," *Nat. Math. Mag.*, vol. 16, no. 1, p. 54, 1941.

[3]. H. B. Keynes and A. M. Olson, "Redesigning the calculus sequence at a research university: issues, implementation, and objectives," *Int. J. Math. Educ. Sci. Technol.*, vol. 31, no. 1, pp. 71–82, 2000.

[4]. R. Rogers, *Putting the differentials back into differential calculus*. Washington, D.C: Mathematics Association of America, 2005.

[5]. S. Salas and E. Hille, *Calculus one and several variables*. New York, NY: John Wiley & Sons, 1990.